RAN-2003000201030034

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F.Y.B.Sc. (Sem. - I) (ATKT) Examination

March - 2023

MTH - 102 Mathematics - II

Time: 1 Hour] **[Total Marks: 50**

સૂચના : / Instructions

(૧)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **F.Y.B.Sc. (Sem. - I) (ATKT)**

Name of the Subject :

☛ **MTH - 102 Mathematics - II**

Subject Code No.: **2003000201030034**

Seat No.:

Student's Signature

(2) આ પ્રશ્નપત્રમાં (૧) થી (૧૬) પ્રશ્નના ૧ ગુણ અને (૧૭) થી (૩૩) પ્રશ્નના ૨ ગુણ છે.
Question (1) to (16) carry ONE mark and (17) to (33) carry TWO marks.

(3) દરેક પ્રશ્નનો ફક્ત એક જ સાચો ઉત્તર છે.
There is only ONE correct answer for each question.

(4) પ્રચલિત સંકેતોને અનુસરો.
Follow usual symbols.

(5) પરીક્ષાનો સમય ૧ કલાક નો છે.
The EXAM is of 1 hour duration.

***O.M.R. Sheet* ભરવા અંગેની અગત્યની સૂચનાઓ આપેલ**
***O.M.R. Sheet*ની પાછળ છાપેલ છે.**
Important instructions to fillup O.M.R. Sheet
are given on back side of the provided O.M.R. Sheet.

SECTION- A / વિભાગ - A

(Question number 1 to 16 each is of 1 mark)

(પ્રશ્ન ક્રમાંક 1 થી 16, દરેકનો 1 ગુણ છે.)

1. If $y = \frac{1}{(3x+2)^6}$ then $y_n =$ _____.

જો $y = \frac{1}{(3x+2)^6}$ હોય તો $y_n =$ _____.

(A) $\frac{(-1)^n (n-5)!2^n}{5!(3x+2)^{n+6}}$ (B) $\frac{(-1)^n (n-5)!3^n}{5!(3x+2)^{n+6}}$

(C) $\frac{(-1)^{n+1} (n-5)!2^n}{5!(3x+2)^{n+6}}$ (D) $\frac{(-1)^{n+1} (n-5)!3^n}{5!(3x+2)^{n+6}}$

2. If $y = \log x$ then $y_n =$ _____.

જો $y = \log x$ હોય તો $y_n =$ _____.

(A) $\frac{(-1)^n (n-1)}{x^n}$ (B) $\frac{(-1)^n (n-1)!}{x^n}$

(C) $\frac{(-1)^{n+1} (n-1)}{x^n}$ (D) $\frac{(-1)^{n-1} (n-1)!}{x^n}$

3. If $y = a^{mx+k}$ then $y_n =$ _____.

જો $y = a^{mx+k}$ હોય તો $y_n =$ _____.

A) $m^n (\log a)^n a^{mx+k}$ (B) $m^n (\log a) a^{mx+k}$

(C) $m (\log a)^n a^{mx+k}$ (D) $m (\log a) a^{mx+k}$

4. If $y = x^4 + 3x^3 + 2x^2 + x + 1$ then $y_4 =$ _____.

જો $y = x^4 + 3x^3 + 2x^2 + x + 1$ હોય તો $y_4 =$ _____.

(A) 4! (B) 4

(C) 3! (D) 3

5. For real valued function f,

$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \frac{h^3}{3!} f'''(a) + \ldots + \frac{h^{n-1}}{(n-1)!} f^{n-1}$

(a)+R;$0 < \theta < 1$ is called _____.

(A) Taylor's expansion (B) Maclaurin's expansion

(C) A and B both (D) None of these

વાસ્તવિક વિધેય f માટે

$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \frac{h^3}{3!} f'''(a) + \ldots + \frac{h^{n-1}}{(n-1)!} f^{n-1}$

(a)+R;$0 < \theta < 1$ ને _____કહેવાય છે.

(A) ટેલર નું વિસ્તરણ (B) મેકલોરીન વિસ્તરણ

(C) A અને B બંને (D) આ પૈકી એક પણ નહિ

6. $f(x) \frac{1}{1+x^2}$ is _____ function in $(-\infty, 0)$

(A) Increasing (B) Constant

(C) Decreasing (D) None of these

$f(x) \frac{1}{1+x^2}$ વિધેય $(-\infty, 0)$ માં _____ છે.

(A) વધતું (B) અચળ

(C) ઘટતું (D) આ પૈકી એક પણ નહિ

7. A real valued function f is continuous on $[a, b]$ and differentiable in $[a, b]$ then there exists $\lambda \in (a,b)$ such that $f'(\lambda) =$ _____.

(A) $\frac{f(b)-f(a)}{b-a}$ (B) $\frac{f(a)-f(b)}{b-a}$

(C) 0 (D) None of these

જો વાસ્તવિક વિધેય f એ $[a, b]$ માં સતત અને $[a, b]$ માં વિકલનીય હોય તો $\lambda \in (a, b)$ એવા મળે જે જેથી $f'(\lambda) =$ _____.

(A) $\frac{f(b)-f(a)}{b-a}$ (B) $\frac{f(a)-f(b)}{b-a}$

(C) 0 (D) આ પૈકી એક પણ નહિ

8. A real valued function f is continuous on $[0, x]$ where $x > 0$ and differentiable in $(0, x)$ then $f(x) - f(0) =$ _____Where $\theta \in (0, 1)$.

(A) $f'(\theta x)$ (B) $xf'(\theta)$

(C) $xf'(\theta x)$ (D) $\theta f'(\theta x)$

જો વાસ્તવિક વિધેય f એ $(0, x)$ જ્યા $x > 0$ માં સતત અને $(0, x)$ માં વિકલનીય હોય તો $f(x) - f(0) =$ _____ જ્યાં $\theta \in (0, 1)$.

(A) $f'(\theta x)$ (B) $xf'(\theta)$

(C) $xf'(\theta x)$ (D) $\theta f'(\theta x)$

9. Curve $y = e^{5x}$ is _____.

(A) Concave downward (B) Concave upward

(C) Convex upwards (D) None of these

વક્ર $y = e^{5x}$ _____ છે.

(A) અધઃઅંતમુખ (B) ઉધર્વ-અંતર્મુખ

(C) ઉધર્વ-બહિર્મુખ (D) આ પૈકી એક પણ નહિ

10. The point of inflexion of $y = 3x^5 - 40x^3 + 3x - 20$ is _____.

(A) $x = 0$ (B) $x = 2$

(C) $x = -2$ (D) All of these

$y = 3x^5 - 40x^3 + 3x - 20$ નું વક્રતા પરીવૃતિ બિંદુ _____ છે.

(A) $x = 0$ (B) $x = 2$

(C) $x = -2$ (D) આ પૈકી બધા

11. Vertical asymptote of the curve $y = \frac{2x-3}{x^2-3x+2}$ are _____.

વક્ર $y = \frac{2x-3}{x^2-3x+2}$ ના લંબક અનંત સ્પર્શકો _____છે.

(A) $x = 1$ and $x = -2$ (B) $x = -1$ and $x = 2$

(C) $x = 1$ and $x = 2$ (D) $x = -1$ and $x = -2$

12. The curvature of the curve $y = \sin x$ at the point $\left(\frac{\pi}{2}, 1\right)$ is _____.

વક્ર $y = \sin x$ ની $\left(\frac{\pi}{2}, 1\right)$ બિંદુએ વક્રતા _____ છે.

(A) 1 (B) 2

(C) 3 (D) 4

13. $\int_0^{\pi/2} \sin^7 x dx =$ _____.

(A) $-\frac{35}{16}$ (B) $\frac{16}{35}$

(C) $-\frac{16}{35}$ (D) $\frac{35}{16}$

14. Justify that $\int_0^{\pi/2} \sin^{10} x dx = \frac{63}{512}$ is true?

(A) Yes (B) No

(C) Can't say anything (D) Undefined

વિધાન $\int_0^{\pi/2} \sin^{10} x dx = \frac{63}{512}$ સાચું છે?

(A) હા (B) ના

(C) કશું જ ન કહેવાય (D) અવ્યાખ્યાયિત

15. $\int_0^{\pi/2} \sin^4 x \cos^3 x \, dx =$ _____.

(A) $\frac{2}{35}$ (B) $\frac{2}{35}\pi$

(C) $\frac{35}{2}$ (D) $\frac{35}{2}\pi$

16. $\int \cos ec^n x dx =$ _____.

(A) $\frac{\cot x \cos ec^{n-2}x}{n-1} + \frac{n-2}{n-1} \int \cos ec^{n-2} x dx$

(B) $\frac{\cot x \cos ec^{n-2}x}{n-1} - \frac{n-2}{n-1} \int \cos ec^{n-2} x dx$

(C) $\frac{-\cot x \cos ec^{n-2}x}{n-1} - \frac{n-2}{n-1} \int \cos ec^{n-2} x dx$

(D) $\frac{-\cot x \cos ec^{n-2}x}{n-1} + \frac{n-2}{n-1} \int \cos ec^{n-2} x dx$

SECTION - B / વિભાગ - B

(Question number 17 to 33 each is of 2 marks)

(પ્રશ્ન ક્રમાંક 17 થી 33, દરેકના 2 ગુણો છે.)

17. If $y = xe^x$ then $y_n =$ _____.

(A) nxe^x (B) $(x + n)e^x$

(C) ne^x (D) None of these

જો $y = xe^x$ હોય તો $y_n =$ _____.

(A) nxe^x (B) $(x + n)e^x$

(C) ne^x (D) આ પૈકી એક પણ નહિ

18. If $y = \sin kx + \cos kx$ then $y_n =$ _____.

જો $y = \sin kx + \cos kx$ હોય તો $y_n =$ _____.

(A) $k^n [1 + (-1)^n \cos 2kx]^{\frac{1}{2}}$ (B) $k^n [1 + (-1)^n \sin 2kx]^{\frac{1}{2}}$

(C) $k^n [1 + (-1)^n \sin kx]^{\frac{1}{2}}$ (D) $k^n [1 + (-1)^n \cos kx]^{\frac{1}{2}}$

19. If $y = \frac{1}{4}\ [e^{2x} \sin x + e^{2x} \sin 3x]$ then $y_n =$ _____.

જો $y = \frac{1}{4}\ [e^{2x} \sin x + e^{2x} \sin 3x]$ હોય તો $y_n =$ _____.

(A) $\frac{e^{2x}}{4}\left[5^{\frac{n}{2}} \sin\left(x+n\ tan^{-1}\frac{1}{2}\right)+13^{\frac{n}{2}}\ \sin\left(3x+n\ tan^{-1}\frac{3}{2}\right)\right]$

(B) $\frac{e^{2x}}{4}\left[5^{\frac{n}{2}} \sin\left(3x+n\ tan^{-1}\frac{3}{2}\right)+13^{\frac{n}{2}}\ \sin\left(x+n\ tan^{-1}\frac{1}{2}\right)\right]$

(C) $\frac{e^{2x}}{4}\left[5^{\frac{n}{2}} \sin\left(x+n\ tan^{-1}\frac{3}{2}\right)+13^{\frac{n}{2}} \sin\left(3x+n\ tan^{-1}\frac{1}{2}\right)\right]$

(D) $\frac{e^{2x}}{4}\left[13^{\frac{n}{2}} \sin\left(x+n\ tan^{-1}\frac{3}{2}\right)+5^{\frac{n}{2}}\ \sin\left(3x+n\ tan^{-1}\frac{1}{2}\right)\right]$

20. If $y = \frac{1}{(ax+b)^m}$; $ax + b \neq 0$ and $m \in N$ then $y_n =$ _____.

જો $y = \frac{1}{(ax+b)^m}$; $ax + b \neq 0$, $m \in N$ હોય તો $y_n =$ _____.

(A) $\frac{(-1)^n\ (m+n)!a^n}{(m-1)!(ax+b)^{m+n}}$

(B) $\frac{(-1)^n\ (m+n)!b^n}{(m-1)!(ax+b)^{m+n}}$

(C) $\frac{(-1)^n\ (m+n-1)!a^n}{(m-1)!(ax+b)^{m+n}}$

(D) $\frac{(-1)^n(m+n-1)!b^n}{(m-1)!(ax+b)^{m+n}}$

21. If $y = \frac{ax}{(ax+b)}$; $ax + b \neq 0$ then $y_n =$ _____.

જો $y = \frac{ax}{(ax+b)}$; $ax + b \neq 0$ હોય તો $y_n =$ _____.

(A) $\frac{(-1)^n\ (n)!a^n\ b}{(ax+b)^{n+1}}$

(B) $\frac{(-1)^{n+1}\ (n)!a^n b}{(ax+b)^{n+1}}$

(C) $\frac{(-1)^n\ n!a^{n-1}b}{(ax+b)^{n+1}}$

(D) $\frac{(-1)^{n+1}\ n!a^{n-1}b}{(ax+b)^{n+1}}$

22. In _____ interval, function $f(x) = 2x^3 - 15x^2 + 36x + 1; x \in R$ is decreased.

(A) $[3, \infty]$ (B) $[-\infty, 2]$

(C) $[2, 3]$ (D) None of these

વિધેય $f(x) = 2x^3 - 15x^2 + 36x + 1; x \in R$ _____ અંતરાલમાં ઘટતું છે.

(A) $[3, \infty]$ (B) $[-\infty, 2]$

(C) $[2, 3]$ (D) આ પૈકી એક પણ નહિ

23. For the function $f(x) = e^x; x \in [0,1]$, according to the Lagrange's theorem's

$\lambda =$ _____.

વિધેય $f(x) = e^x; x \in [0,1]$ માટે લાગ્રાન્જના પ્રમેય અનુસાર $\lambda =$ _____.

(A) $e - 1$ (B) $\frac{1}{e-1}$

(C) $\log\left(\frac{1}{e-1}\right)$ (D) $\log(e - 1)$

24. Which of the following is true for $0 < a < b$?

નીચેનું કયું $0 < a < b$ માટે સત્ય છે?

(A) $\frac{a-b}{a^2+1} < \tan^{-1} a - \tan^{-1} b < \frac{a-b}{b^2+1}$

(B) $\frac{a-b}{b^2+1} < \tan^{-1} a - \tan^{-1} b < \frac{a-b}{a^2+1}$

(C) $\frac{b-a}{a^2+1} < \tan^{-1} a - \tan^{-1} b < \frac{b-a}{b^2+1}$

(D) $\frac{b-a}{b^2+1} < \tan^{-1} a - \tan^{-1} b < \frac{b-a}{a^2+1}$

25. For which value of λ, function $f(x) = \log x$; $x \in [1,e]$ satisfy $f'(\lambda) = \frac{f(e) - f(1)}{e-1}$?

જો $f(x) = \log x$; $x \in [1,e]$ તો (λ) ની કઈ કિમંત માટે $f'(\lambda) = \frac{f(e) - f(1)}{e-1}$ થાય?

(A) $e-1$ (B) $\frac{1}{e-1}$

(C) $e+1$ (D) $\frac{1}{e+1}$

26. The curvature of the curve $y = \log_e x$ at point (1, 0) is _____.

વક્ર $y = \log_e x$ ની (1, 0) બિંદુએ વક્રતા _____ છે.

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) $2\sqrt{2}$ (D) $\frac{1}{2\sqrt{2}}$

27. Curve $y = \cos x$; $x \in (-2\pi, 2\pi)$ becomes concave upward in _____. interval.

$y = \cos x$; $x \in (-2\pi, 2\pi)$ વક્ર _____ અંતરાલમાં ઉર્ધ્વ અંતર્મુખ બને છે.

(A) $\left(\frac{3\pi}{2}, 2\pi\right)$ (B) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(C) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ (D) $\left(-2\pi, -\frac{3\pi}{2}\right)$

28. Asymptote of the curve $y = \frac{x^2 + 2x - 1}{x}$ is _____.

વક્ર $y = \frac{x^2 + 2x - 1}{x}$ નો અનંત સ્પર્શક _____છે.

(A) $y = 2x + 1$ (B) $y = 2x-1$

(C) $y = x-2$ (D) $y = x + 2$

29. The point of inflexion of $y = x^3 - 6x^2$ is _____.

$y = x^3 - 6x^2$ નું વકતા પરીવૃતિ બિંદુ _____છે.

(A) (1, 2) (B) (–l, 2)

(C) (–l, –2) (D) (l, –2)

30. $\int \tan^4 x dx =$ _____.

(A) $\frac{\tan^3 x}{3} - \tan x + x$ (B) $\frac{\tan^3 x}{3!} - \tan x + x$

(C) $-\frac{\tan^3 x}{3} + \tan x - x$ (D) $-\frac{\tan^3 x}{3!} + \tan x - x$

31. $\int_0^{\pi/4} \sin^5 2x dx =$ _____.

(A) $\frac{4}{15}\pi$ (B) $\frac{4}{15}$

(C) $\frac{15}{4}\pi$ (D) $\frac{15}{4}$

32. $\int \cos ec^4 x dx =$_____.

(A) $-\frac{\cot x \cos ec^2 x}{3} + \frac{2}{3}\cot x$

(B) $-\frac{\cot x \cos ec^2 x}{3} - \frac{2}{3}\cot x$

(C) $\frac{\cot x \cos ec^2 x}{3} + \frac{2}{3}\cot x$

(D) $\frac{\cot x \cos ec^2 x}{3} - \frac{2}{3}\cot x$

33. $\int x^2 \sin 2x dx =$ _____

(A) $\frac{x^2 \cos 2x}{2} + \frac{x \sin 2x}{2} + \frac{\cos 2x}{4}$

(B) $-\frac{x^2 \cos 2x}{2} + \frac{x \sin 2x}{2} + \frac{\cos 2x}{4}$

(C) $\frac{x^2 \cos 2x}{2} - \frac{x \sin 2x}{2} + \frac{\cos 2x}{4}$

(D) $-\frac{x^2 \cos 2x}{2} - \frac{x \sin 2x}{2} + \frac{\cos 2x}{4}$

SPACE FOR ROUGH WORK